



Reed field $\rightarrow \text{Form } \subset \mathbb{CP}^3$.

no # #

Hodge field
(2010)

Hopf algebra

modules
 $a \circ b \in \text{Hopf algebra}$
 $= b \circ a$

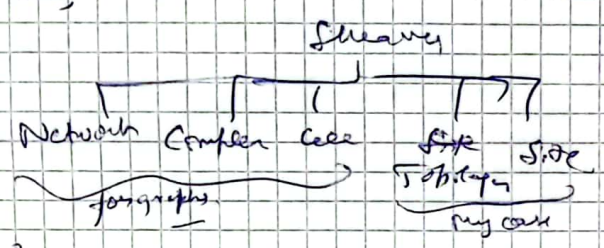
$$H^i(kG, A) \simeq H^i(G, A^*)$$

$$\text{Hom}(C, A)$$

If A is an UL -module, then for $i \geq 1$,

$$H^i(UL, A) \simeq H^i(L, A^*)$$

(1) Cellular sheaves



$$C^0(f) \rightarrow C^1(f) \rightarrow C^2(f)$$

Graphs
Laplacian

Vertex
data

Edge
data

2-cell
data

3-cell
data

$$H^0(f)$$

$$H^1(f)$$

Hodge
Laplacian



For X a graph,
if the constant sheaf
 $\mathbb{C}$ is the graph Laplacian

Hodge
sheaves

$$H^k f = \text{Res } L^k$$

(2) Tarski Laplacian

Tarski cohomology; $H^n f = (\text{Id } f)$

$\rightarrow$ Evid Problem (#735) Ben Morrell

Stationary soln of Euler eqn

if volume preserved $x \mapsto y$ on $M = S^3/\mathbb{Z}_2$

Hydrodynamic	Symplectic	Regularly	Always - periodic orbits
1) arbitrary Impairing geometric constraint $\star UH$ (def free)		C^1 C^0	No Schweitzer '94 No (Kupertsov '94)
2) Volume preserving $L_x vol = 0$ $d(i_x vol) = \omega$ UH (def free)	Hamiltonian $\omega \neq 0$ closed $L_x \omega = 0$		No (G... '96)
2) Euler $\nabla_X X = -\nabla P$ $d = g(X, \cdot)$ $h = P + \frac{1}{2} X ^2$	$i_X d = -dh$ $d(X) > 0$	C^0 (Geddes) C^1	Yes (E...-G... '00)
Beltrami fields $\nabla X = \lambda X$	(h, d) Strongly non-degenerate $i_X d = 0, d(X) > 0$		Yes Hitchcock-Talbot '09
UH 	d contact X Reeb	C^0	Klofer '93 in S^3 Taubes '93 in H^3

Obstruction to Euler: Reeb comp.

Wick's plug

$$0 = \int_A dd = \int_{\partial A} d > 0$$

One change any vol + pres X by

inserting a plug with a Reeb component

keeping it v.t. + pres



Assume:

$$i_X d = -dh$$

$$d(X) \geq 0$$

$\Rightarrow h$ integral of motion

$\Rightarrow h = \text{constant}$

$$\Rightarrow i_X d = 0$$

(2) Given X^1 , $i_X d\alpha = -d\alpha$
 $d\alpha > 0$

(3)

Want $\rightarrow i_X d\alpha = 0$
 $r(x) > 0$

* Bernoulli's method (free): Result &
 Bernoulli function $\rightarrow$ (Penalty-Sales $\rightarrow$ pay)
 Ideas Assume $h_0(\omega)$

Closed 1-form: $(d\alpha = 0)$

Construct near critical level sets of h

* Integrals are like degree of freedom!

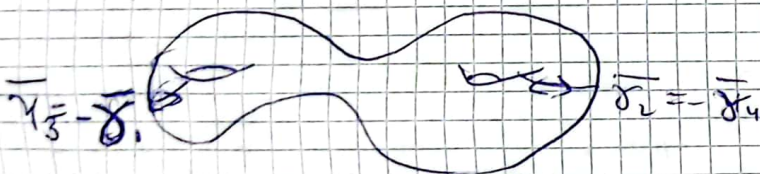
away from crit. level sets, foliation by invariant tori,

$\mathbb{R}^2 \rightarrow \mathbb{R}^2$ flow is linear.

Why does it not work in smooth case? (counterexamples in ∞ case)
 discrete subgroup $\mathbb{Z}^n$.
 closed diff. surface

$\Sigma = M / \sim$
 Canonical 1-form
 $(S^1 \times \Sigma, d)$ contact, Reeb vector field (R)

* Look at all other counterexamples related to smooth solutions



Let γ be closed Reeb orbits $(\gamma_1, \dots, \gamma_n)$

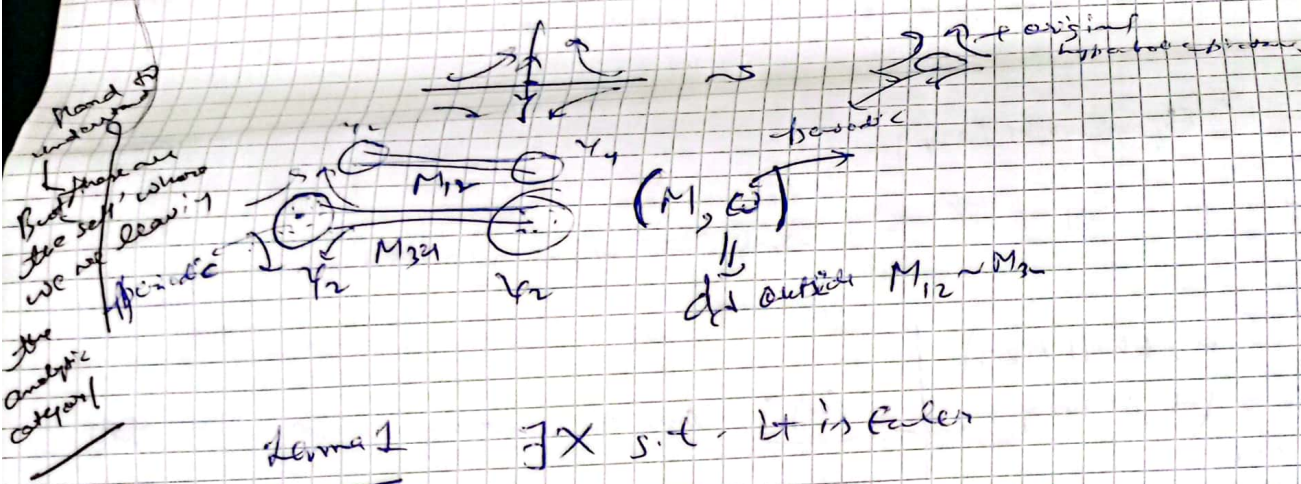
$$\gamma_1 \int d\alpha = \gamma_2 \int d\alpha = \gamma_3 \int d\alpha = \gamma_4 \int d\alpha = \dots$$



Fabricado en España

Modify d on original V_i or V_i , to d' instead

(7)



Lemma 1

$\exists X$ s.t. it is Euler

Lemma 2

$\exists \tilde{X}$ s.t. it is not Euler

($\Rightarrow$ not Beltrami)

Q

If every subsurface of X is Euler,

(Tao)

is X also Beltrami?

Take $X \sim \tilde{R}$ outside

$$M_{12} \cup M_{34} = [3, 4] \times T^2$$

$$\cong [1, 2] \times T^2$$

$$\text{s.t. } X = d_2$$

$$\sigma \rightarrow \psi, z$$

$$\text{on } d \in \mathbb{H} \times T^2$$

$\Rightarrow \tilde{X}$ does not extend over $M_{12} \cup M_{34}$ as Euler: $\int_{\tilde{X}} \omega = \int_{\tilde{X} \cap \partial} \omega = \int_{\partial} \omega = \int_{\partial} \omega = \int_{\partial} \omega = \int_{\partial} \omega$

$$\chi(\tilde{X}) > 0; \int_{\tilde{X}} \omega = -dh, \quad \int_{\tilde{X}} \omega = -dh$$

$$\int_{\gamma_2} \omega - \int_{\gamma_1} \omega = \langle (T_2 - T_1) + \langle \beta, [\gamma_2] - [\gamma_1] \rangle \rangle$$

$$0 = \int_{\gamma_4} \omega - \int_{\gamma_3} \omega = \langle (T_2 - T_1) + \langle \beta, [\gamma_4] - [\gamma_3] \rangle \rangle$$

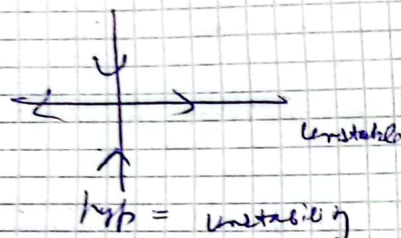
$$0 = 2 \langle (T_2 - T_1) + \langle \beta, [\gamma_2] + [\gamma_4] \rangle \rangle - \langle [\gamma_1] + [\gamma_3] \rangle$$

(mj (Birkhoff 1944))

Let $f \in \text{Sym}^2(\mathbb{R}^2)$ s.t. $\# \text{fix } f = \# \text{f.p.} = 2$.

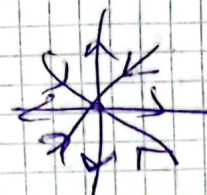
(mj (Birkhoff 1927))

formal stability $\neq$ stability $\leftarrow$ Conj.: There are unstable elliptic fixed pts.



$$H(x,y) = xy$$

$$z \mapsto z^2$$



$$\tilde{H} = H(z^q)$$

$$\mathbb{R}_{p/q} \circ \phi_{\tilde{H}}$$

maybe stable?

* A fixed point is elliptic if its eigenvalues are on

$|S|=1$ with irrational argument.

* A fixed p is unstable if $\exists$ open $U \ni p$, $\forall$ open $V \ni p$, $U \not\subset \bigcap_n f^n(V)$

Theorem Arnold - Kato 1970:

There are ergodic smooth symplectic of the cylinder or torus sphere

without periodic or too periodic points

(important slides)

Theorem (Nevai - Nirenberg):

If $(I_n)_n$ is a sequence of complex structures on $\mathbb{R}^2$,

which C^∞ -converges to I_∞ , then

I_∞ is complex structure, and if

$(\Omega_n)_n$ is seq. of I_n -holomorphic extensions of Ω which converges to Ω_∞ , then Ω_∞ is a I_∞ -holomorphic extension of Ω .



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→ Contraction (or Convergence of sequence) method for 6
 automorphism group (and related extensions) → see exercise in p. 15 of
 Mean approx. theorem [Pierre Berge]

3rd fall (Ans)
 →

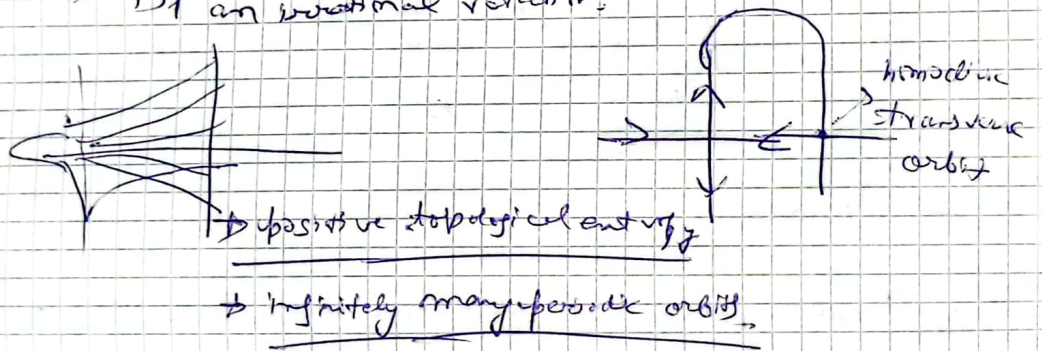
C^∞ -generic Reeb flows & hyperbolicity,

M-closed oriented 3-manifold ($\mathbb{S}^3$)

Defn: A periodic orbit is non-degenerate if λ is never an eigenvalue
 of DP^n , $\forall n$.

$\bar{X}$ is volume preserving, two types non-deg. of orbits

- hyperbolic (real EVs) → unstable and stable manifolds
- elliptic → Df an irrational rotation.



Thm. (CDHR)

C^∞ -generic Reeb vector fields have homoclinic orbits for all
 hyperbolic periodic orbits.

Hypothesis X is Reeb vector field — strongly non-degenerate
 - periodic orbits are equidistributed → Chromological metric?

X is Reeb, there is a contact form α ,



Art: $\alpha(X) = 1$, $L_X \alpha = 0$

$\alpha \wedge d\alpha > 0$ is an invariant volume form.

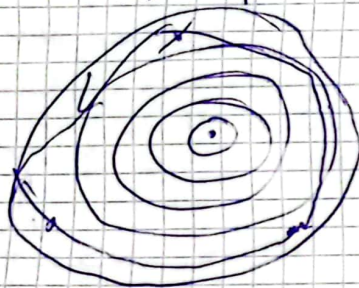
C^∞ -generic Reeb vector fields (Ivrii '23) are ambient

(contact topology) Δ

— Zehnder's condition

Near an elliptic periodic orbit

check more about it later!



— Remark

Implies that hyperbolic periodic orbits are dense in M .

Theorem (Conzatas, Oliveira 22 (preprint))

[(Σ, g) a Riemannian surface,
 C^∞ -generically on (g) every hyperbolic
periodic orbit has a transverse homoclinic
of the geodesic flow]

Thm (CDHR, Forster - Mazzeuchi)

A C^∞ -generic Reeb vector fields admits a Birkhoff section.
 Birkhoff section
 (Forster - Mazzeuchi) strongly non-degenerate

• (CDHR) non-degenerate + equivalent

A Birkhoff section
 (M^3, X) a Birkhoff section

$S \rightarrow M$ immersed surface

(S) is a collection of periodic orbits of X .

The other ingredients:

Thm (Le Calvez, Samborski '23)

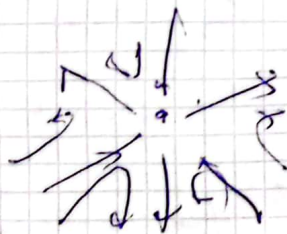
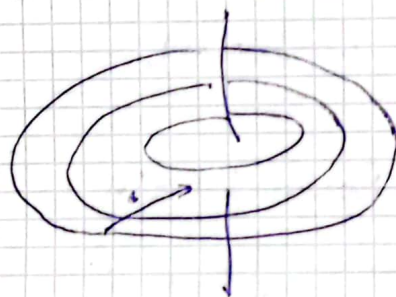
S is a closed surface

$f: S \rightarrow S$ an area preserving C^1 -diffeomorphism

→ f non-degenerate

→ number of hyperbolic points $\geq \max(0, 2g - 2)$

⇒ every hyperbolic orbit has a homoclinic.



Q A Reeb vector field like this without elliptic periodic orbits,
How far is it from being Anders?

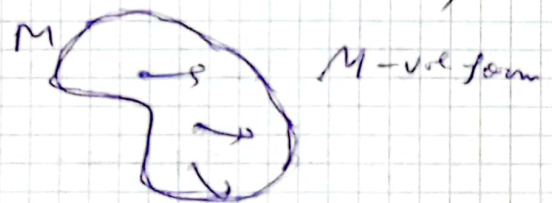
Corollary: of the main theorem

When the given hypothesis, the Reeb vector field admits
on embedded bi-foliated surfaces (up to adding lots of boundary
components).

→ Boris Khosin (Geodesics, vortex sheets, and generalised fluid
flow)

$$\int_{\Sigma} \nabla \cdot V + \nabla \cdot V = -\nabla \cdot p$$

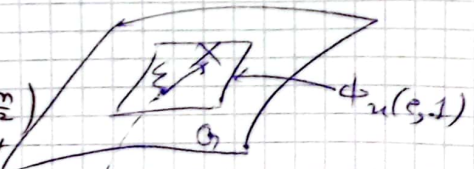
$\operatorname{div} V = 0$; V is tangent to ∂M .



Theorem (Arnold 1966)

$G = \operatorname{Diff}_\mu(M)$ of volume-preserving diffeomorphisms
w.r.t. the right-invariant L^2 -metric

Suitable functional-analytic setting of Sobolev H^s (for $s > 1 + \frac{n}{2}$)
($S^1, \operatorname{SO}(3)$) $\rightarrow \mathbb{R}^k \rightarrow$ Helson-Seyg and some vector
magnetic chain.



0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99

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Score: 22/25

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1. **Identify the problem.** The first step in the problem-solving process is to identify the problem. This involves recognizing the issue, gathering information, and defining the problem clearly.

4. How many people

\_\_\_\_\_

$$0 \leq \beta_1 < \beta_2 < \dots < \beta_{n-1} < \beta_n = 1$$

\_\_\_\_\_

guyana b. a. p. m. b. a. n.

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1995 = 100

1. **Introduction**  
 2. **Background**  
 3. **Methodology**  
 4. **Results**  
 5. **Discussion**  
 6. **Conclusion**  
 7. **References**  
 8. **Appendix**  
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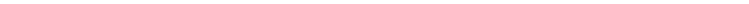
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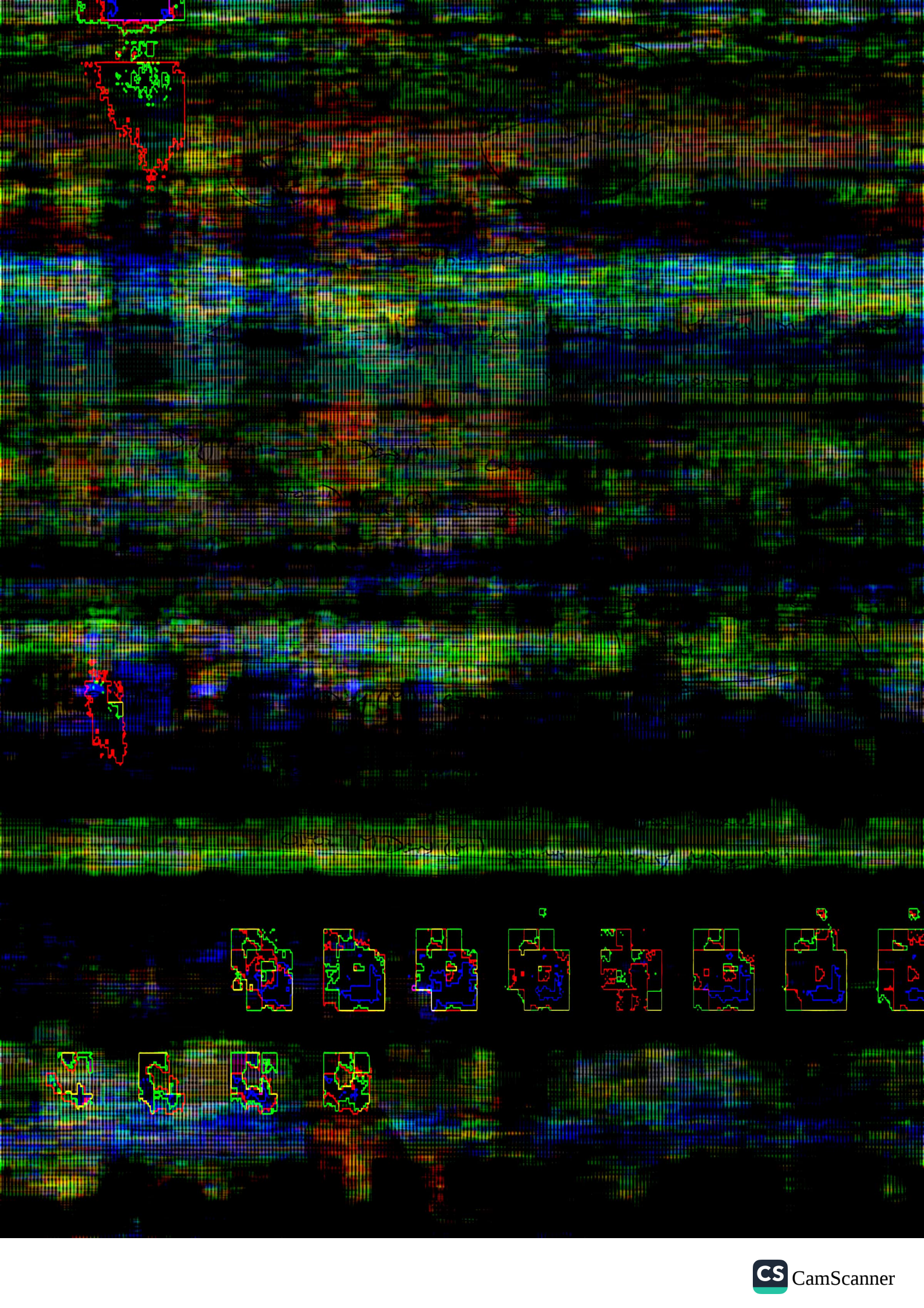
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$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

is in normal di confinement of a

Figure 1. The effect of the number of trials on the number of correct responses. The number of correct responses was significantly higher than the number of incorrect responses in all cases. The number of correct responses was significantly higher than the number of incorrect responses in all cases.





Candabai (17/9/20)  
 → Elgindi (2021)

Elgindi & Parag (2020)  $C^{1,2}$  (with surface)

Ans-sym. Euler without surface  $\rightarrow$

$$w_t + u^x w_x + u^z w_z = u^x w_y$$

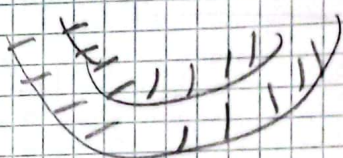
Placing bumps in outward-inward direction

Step 1: Find a mech. for growth

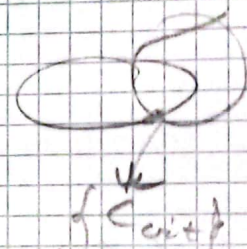
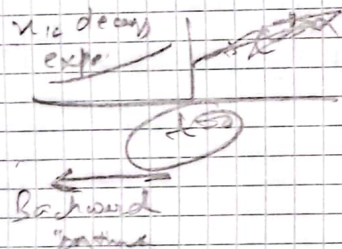
Infinite no of bumps, each bump is comp. supported

$k^{th}$  bump

$$w = w_{int} + \sum_{j \in I} w_j + \sum_{j \in O} w_j$$



CODE:



Dr. C, L. Moroz-Zemba (2021)

$\frac{1}{2}u$   $\{ C^{1,2-\epsilon} \cap L^2 \}$  force

$$w \approx A \psi(Lx_1) \psi(Lx_1 - k(w_3)) \sin Mx_2 \sin(Lx_1) e^{i \dots}$$

$\rightarrow$  to make it all right

$$w = \sum_{n=1}^{\infty} w_n(x_1, t)$$

How:

Riesz transform:

$$R: \mathcal{S}'(\mathbb{R}^n) \rightarrow \mathcal{S}'(\mathbb{R}^n)$$

Eggen (2021):

$C^\infty$  vanishing, for small  $\alpha$ .

3) Euler singularity, for asymptotic Euler  
 $(\omega_t + u \omega_x = u_x \omega, \text{ if } \omega \sim e^{u^2})$ .

How  $u, \omega$ :

$$u_1 = \frac{u^0}{\gamma}, \quad \omega_1 = \frac{\omega^0}{\gamma}, \quad \psi_1 = \frac{\psi^0}{\gamma}$$

$$u_0^0(\gamma, z) = \gamma u_0(z), \quad \omega_0^0(\gamma, z) = \gamma \omega_0(z)$$

Lyapunov:  $(u_{1,z})^2 + \omega_1^2$ ; and show that

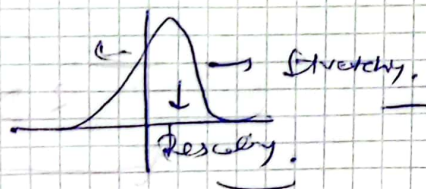
$$\left( (u_{1,z})^2 + \omega_1^2 \right)_t + (u_{1,z}^2) \cdot \partial_z \left( (u_{1,z})^2 + \omega_1^2 \right) = 0$$

Th 1 (Cao-Hou, 2022, 2023):

- 1) Set up into data  $(\psi_0, \omega_0)$
- 2) Comparison with nonlinear Schrödinger eqn. (~~analytic~~ explicit ground state).
- 3) Comparison ansatz to obtain sharp upper bounds for our stability constants.   
  $\rightarrow$  look it up!

2D Boussinesq eqn. in  $\mathbb{R}_+^2$ :

$\rightarrow$  "breaching term" and "scaling term".  
 $(\psi, \theta, \theta)$  and  $(\omega(r), \omega)$



General approach to nonlinear vorticity:

$$v = (0, 0)^T; \text{ write:}$$

$$\frac{dv}{dt} = F(v)$$

$$\text{and } \|F(v)\| = \epsilon \ll 1.$$

$$v = \bar{v} + \tilde{v}; \text{ eqn. for } \tilde{v} \text{ is}$$

$$\frac{d\tilde{v}}{dt} = F(\bar{v} + \tilde{v}) = \underbrace{F(\bar{v})}_{\text{linear}} + \underbrace{\tilde{v}^T \nabla^2 F(\bar{v}) \tilde{v}}_{\text{nonlinear}} + F(\tilde{v}).$$

$$\tilde{v} = \int_0^t e^{(t-s)A} (N(\tilde{v}) + F(\tilde{v})) ds$$

Assume,  $\|e^{tA}\| \leq e^{-\lambda t}$ ,  $\lambda > 0$ ,  
and nonlinear term is bounded as:

$$\|N(\tilde{v})\| \leq C\|\tilde{v}\|^2$$

$$\text{Then we have } \|\tilde{v}(t)\| \leq \int_0^t e^{-\lambda(t-s)} (C\|\tilde{v}(s)\|^2 + \epsilon) ds$$

$$\text{If } 4C\epsilon < \lambda^2, \exists \delta > 0, \text{ s.t.}$$

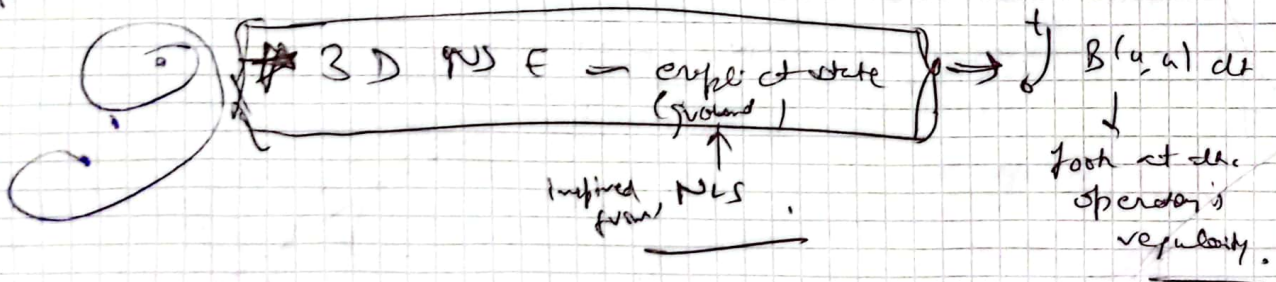
$$C\delta^2 + \frac{\epsilon}{\lambda} \leq \delta$$

Bootstrapping: Suppose  $\|\tilde{v}(s)\| \leq \delta, \forall s \in [0, t]$ , then

$$\|\tilde{v}(t)\| \leq \int_0^t e^{-\lambda(t-s)} (C\delta^2 + \epsilon) ds$$

$$\Rightarrow \|\tilde{v}(t)\| \leq \delta \text{ if time given } \|\tilde{v}(s)\| \leq \delta \leq \frac{C\delta^2 + \epsilon}{\lambda} \leq \delta$$

Not esqr.

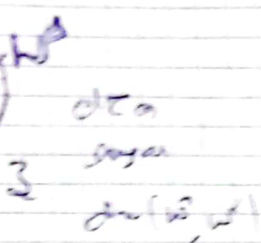


# Proven:

1. find a process approximate (unstable) self-similar etc.

2. Decompose the Blue into explicit part + finite dim.

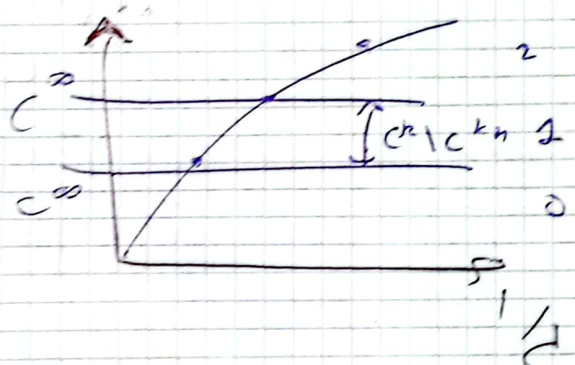
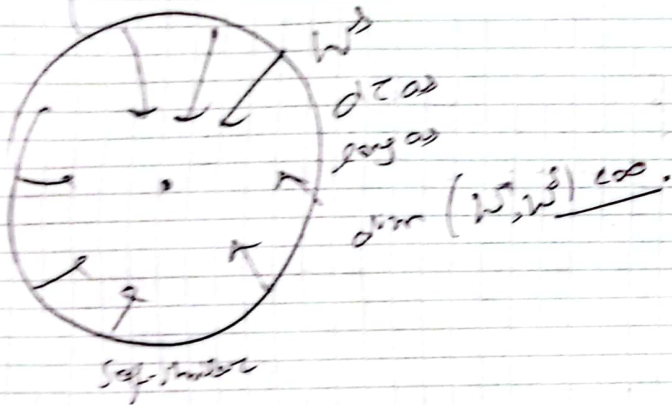
matrix + a tail that can be understood by hand as bounded (using relative compactness).



3. show that it is stable modulo a finite no. of directions, assuming the rest is h. of large dim.

4. one can add some viscosity as long as it is exp. small in self-similar coordinates.

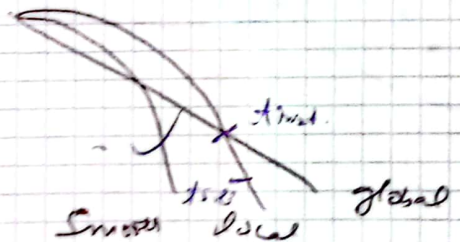
5. Prove non-linear stability.



The center for  $w, z$  reads

$$\frac{\partial}{\partial z} w = \frac{N_w(w, z)}{D_w(w, z)}, \quad \frac{\partial}{\partial w} z =$$

WDE  
System





Prop.:  $\{ \text{exact fields of contact type "convex"} [\{ \} ] \}$  is a  $\mathcal{C}^0$ -open set in  $\mathcal{X}_{\text{cl}}^0(M)$ .

Defn: An involutive condition  $V_0 \in \mathcal{X}_{\text{cl}}$  is of contact type if  $\text{curl } V_0$  is of contact type.

Remark: If  $V_+$  is a soln to Euler, then  $V_+$  is of contact type  $V_+$  if  $\text{curl } V_+ = \left( \frac{2}{\lambda} \right)_{V_+} \text{curl } V_0 \Rightarrow \begin{cases} \text{Lip}_+^\lambda d\beta_0 = d\beta_+, \text{ where } \\ g_+^\lambda H = H \\ \beta_+ = g(V_+, \cdot), \text{ so if } \\ d \text{ is contact and } \\ d\alpha = d\beta_+ \\ \Rightarrow d(g_+^\lambda \alpha) = d\beta_+. \end{cases}$

Corollary:  $\mathcal{L}_{a,h,e}(\mathcal{Z}) = \mathcal{Z}(\mathcal{Z} \cap \mathcal{V}_{a,h,e})$  is  $\mathcal{C}^1$ -open in  $\mathcal{V}_{a,h,e}$ .

Remark:  $\mathcal{L}_{a,h,e}(\mathcal{Z})$  might be empty, it's not necessarily non-empty.

$$\mathcal{Z} \oplus \mathcal{Q} = \text{T.M.}$$

rem (S. Terepis & Kizan '21):

$\mathcal{C}$ -open set

1.  $\forall a, h \neq 0, e \in (0, \infty), \exists V_1, V_2 \in \mathcal{V}_{a,h,e}, \text{ s.t. } V_1 \text{ is contact type, } V_2 \text{ are not contact type.}$

2. Under a technical assumption on  $a$ , given  $\mathcal{Z}$  s.t.  $\mathcal{Z} \in \mathcal{A}_{2, \text{T.M.}}$ ,  $\exists$  functional  $\mathcal{C}: \mathcal{L}_{a,h,e}(\mathcal{Z}) \rightarrow \mathbb{R}$ .

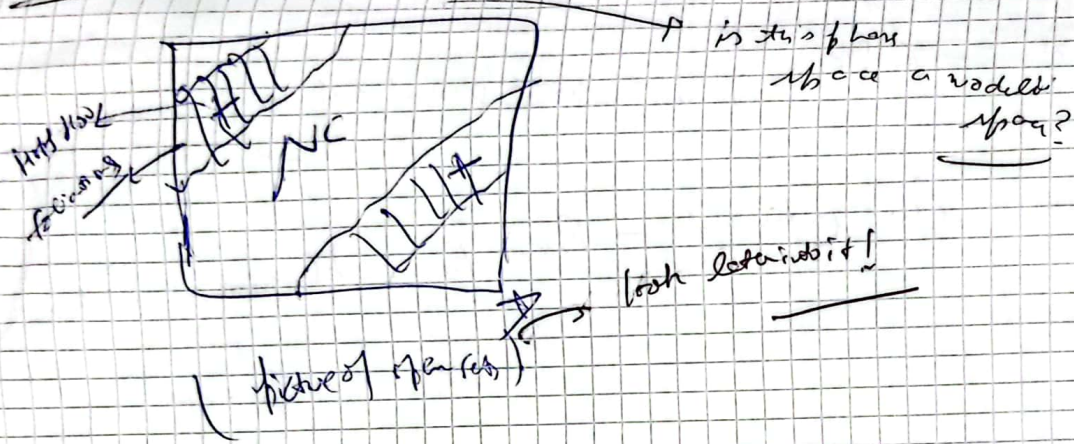


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Fabricado en España

(1) Invariant quantity, continuous with  $\mathbb{R}^3$  topology, non-trivial  
 $\forall e \in \mathbb{R}^3$

Picture:  $\mathcal{V}_{a,h,e}$  in  $(S^3, \text{general})$ ,  $a = \Gamma(\mu, \mu)$



proof: Rank:  $c'$  is obtained from  $\Gamma(\mu, \mu)$

1. Step 1:  $\forall a, h \neq 0$ , constant,  $y_1, y_2 \in \mathbb{R}^3(M, \mu)$ ,

$y_1, y_2 \in a$ , label of  $y_1$

and  $y_1$  is of critical type

Yin-Yang of critical type  $\mathbb{R}^3$  volume

look into it

(Circle class's obstruction to stability of orbit)

2. Step 2:  $\forall c > e_0(a, h)$ , find  $y_1, y_2$ , stability of orbit

(1) Given a constant  $\alpha \sim \in C^k(M, \alpha) \subseteq FCM(M, \mathbb{Z}_2)$ , there?

$\mathbb{F} \tilde{C} = \mathbb{C}^{\infty}(y_1)$ , fluctuating + random

( $\phi \mapsto \tilde{C}(\phi, \alpha)$ )

properties  
 (i)  $\mathbb{C}^{\infty}$ -continuous  
 (ii) often depends only on  $d(\phi)$   
 (iii) non-trivial

$C(v\alpha) = C(\alpha)$  ; for some primitive  $\alpha\beta$

(iii) Abbondando - Bramham - Hyman - Salom  
↓  
(nonlinear)

~~FR~~ David

Eulerian versus Lagrangian perspective (2D Euler).

flow map  $\chi$

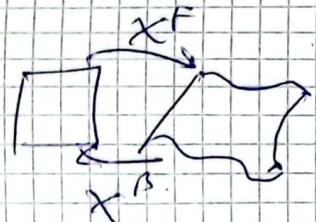
$$\chi: D \times \mathbb{R}^+ \rightarrow D$$

$$u: D \rightarrow \mathbb{R}^2$$

↓  
divergence

$$(D \cdot u) = 0$$

$$\det(D\chi) = 1$$



$$X^B = (X^F)^{-1}$$

$$\int_t \frac{d}{dt} (u \cdot \nu) g = 0$$

$$g(x, t) = g_0(x)$$

method of characteristics

Looking for:

$$\chi^B(\gamma(t), t) = \gamma_0 \text{ for all curves } \gamma \in \mathcal{H}_1$$

(preservation)?

Properties of  $\chi$ :

$$\chi_{[t_1, t_2]} \circ \chi_{[t_0, t_1]} = \chi_{[t_0, t_2]}$$

$\chi(\cdot, t)$  is a one-parameter semi-group.

If  $t_1 < t_2$ , we have forward map.

If  $t_1 > t_2$ , backward map.

2D comp. Euler:

$$\frac{d}{dt} u + (u \cdot \nabla) u = -\nabla p$$

$$\begin{aligned} \nabla \cdot u &= 0 \\ u(x, t=0) &= u_0(x) \end{aligned}$$

$$\boxed{\omega = \nabla \times u}$$

$$\frac{d}{dt} \omega + (u \cdot \nabla) \omega = 0$$

$$\nabla \cdot u = 0$$

$$\boxed{\mu = -\delta^{-1}(\nabla \times \omega)}$$

Bijectivity  
forward  
backward

$$\boxed{D\omega = D\omega_0 + D\chi(x, t)}$$

$$D\chi_{[t, 0]} = D\chi_{[t_1, 0]} \circ D\chi_{[t_2, t_1]}$$

3D Euler:  $\int_t \frac{d}{dt} \omega + (u \cdot \nabla) \omega = (\omega \cdot \nabla) \omega + \nabla \times (\omega \times u)$

Vorticity stretching

Numerical

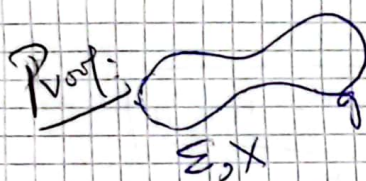
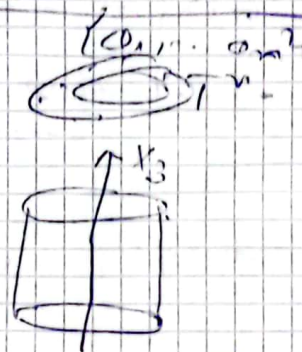
$$(\int_t u + u \cdot v) \propto \beta - \alpha$$

$$(\omega(x)) = (v^{(k)})^{-1} \omega_0(v^{(k)})$$

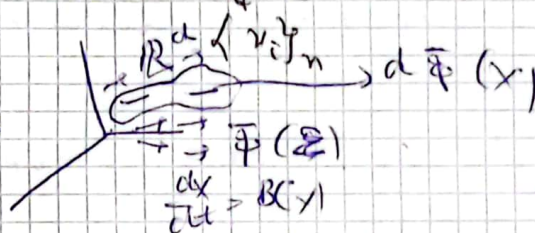
Where,  $u = -\Delta^{-1}(v \times \omega)$   $\det(v^{(k)}) = 1$   
 $\rightarrow$  Numerical

$$\rightarrow (\phi_1, \phi_2, \dots, \phi_m) \in T^m$$

$$\frac{d\phi_i}{dt} = \omega_i \quad \text{along } \gamma$$



$$\xrightarrow{\bar{\Phi}}$$



$$B(x) = d_{\bar{\Phi}(\Sigma)} \bar{\Phi}(x)$$

$$\Delta \Psi_i = d\Psi_i$$

For large  $n$ , this is an embedding

Lemma:  $\bar{\Phi}(p) = (\psi_1(p), \psi_2(p), \dots, \psi_n(p))$   
 $\Sigma \rightarrow \mathbb{R}^n$



Fabricado en España

~~FA~~ Differential Equations

Eulerian versus Lagrangian perspective (2D Euler).

Flow map  $\chi$

$$\chi: D \times \mathbb{R}^+ \rightarrow D$$

$$u: D \rightarrow \mathbb{R}^2$$

↓  
clear

$$(D \cdot u) = 0$$

$$\det(D\chi) = 1$$



$$\int_t \varphi_t(u \cdot \nabla) q = 0$$

$$\varphi(x, t) = \varphi_0(u)$$

method of characteristics

Looking for:

$$\chi^B(\gamma(t), t) = \gamma_0 \text{ for all curves } \gamma(t),$$

(preserved)?

Properties of  $\chi$ :

$$\chi_{[t_1, t_2]} \circ \chi_{[t_0, t_1]} = \chi_{[t_0, t_2]}$$

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2D comp. Euler:

$$\partial_t u + (u \cdot \nabla) u = -\nabla p$$

$$\nabla \cdot u = 0$$

$$u(x, t=0) = u_0(x)$$

$$\omega = \nabla \times u$$

$$\partial_t \omega + (u \cdot \nabla) \omega = 0$$

$$\nabla \cdot \omega = 0$$

$$\mu = -\delta^{-1}(\nabla \times \omega)$$

But -  
Semi-  
local

$$D\omega = D\omega_0 + D\chi(\gamma, t)$$

$$D\chi(\gamma, t) = D\chi_{[t_1, t_2]} \circ D\chi_{[t_2, t_1]}$$

3D Euler:

$$\partial_t \omega + (u \cdot \nabla) \omega = (\omega \cdot \nabla) u + \nabla \times (\omega \times u)$$

Vorticity stretching

Numerically

$$\left( \int_t u + u \cdot \partial \right) \chi_B = \omega$$

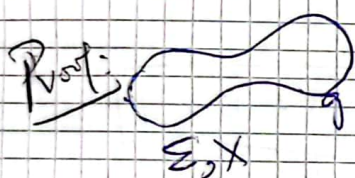
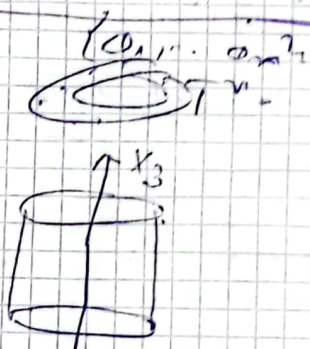
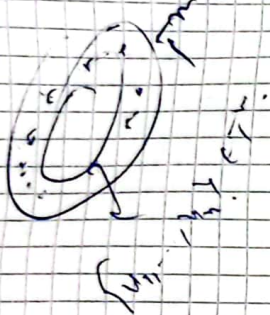
$$\left( \omega(x) = (v^B)^{-1} \omega_0(x^B) \right)$$

Where,  $u = -\Delta^T(v \times \omega)$ ;  $\det(v^B) = 1$ .

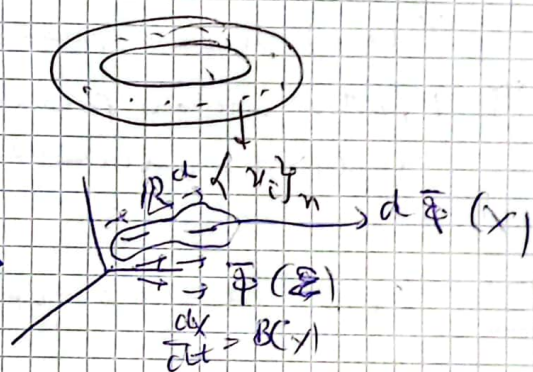
→ Numerically

$$\rightarrow (\phi_1, \phi_2, \dots, \phi_m) \in T^m$$

$$\frac{d\phi_i}{dt} = \omega_i \quad \text{along } \gamma$$



$$\xrightarrow{\bar{\Phi}}$$



$$\Delta \Psi = d \circ \Psi$$

$$B(x) = d_{\bar{\Phi}(x)} \bar{\Phi}(x)$$

For  $\gamma \in \Sigma$ , this is an embedding

Lemma:  $\bar{\Phi}(p) = (\psi_1(p), \psi_2(p), \dots, \psi_n(p))$   
 $\Sigma \rightarrow \mathbb{R}^n$



Fabricado en España

$$\chi(p) = \chi_{ij}^{\mu} c_{\mu}(p)$$

$$d_p \Phi(x) = \sum_{j=1}^N \sum_{\mu} \chi^{\mu}(p) \frac{\partial \psi_j(p)}{\partial x_j}$$

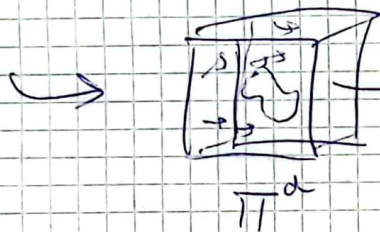
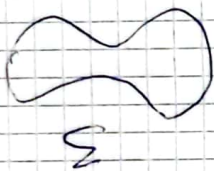
★ Embedding  $\chi(p)$

$$\chi^{\mu}(p) \sim \sum_{\alpha} a_{\alpha} u_{\alpha}(p)$$

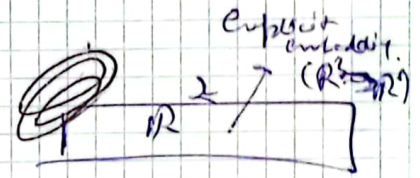
$$\psi_j(p) \sim \sum_{\alpha} b_{\alpha} \psi_{\alpha}(p)$$

$$\frac{\partial}{\partial x_j} = \sum_{i=1}^N \sum_{\mu} (a_{\mu}^i b_{\mu j}) \underbrace{u_{\mu}(p)}_{\chi_{\mu}} \underbrace{\psi_j(p)}_{u_j}$$

# Classical embeddings:



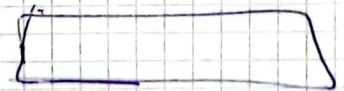
$\mathbb{R}^N$   
Global



→ Area flows:

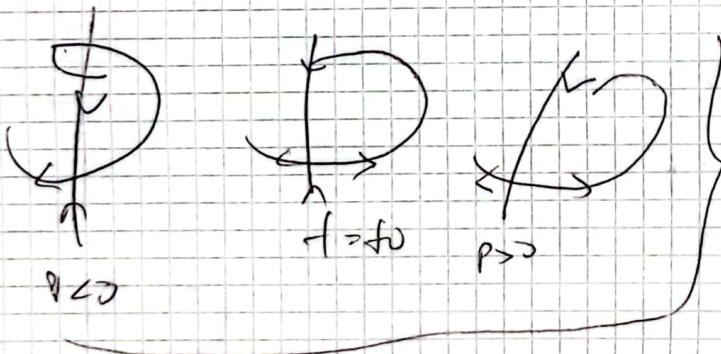
Volume-preserving flow originated to a Birkhoff flow

Local



Measure

$$d\mu \rightarrow \frac{f(x)}{f(y)}$$



★ / Nablachione, but just around me.

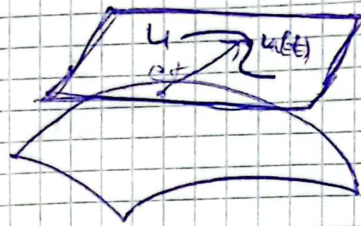
In this setting, I have shown that heteroclinic orbits

$$H \hookrightarrow M(S, M) \hookrightarrow G$$

$$\uparrow$$
  

$$Dy(S)$$

$$X_f \in \mathcal{X}_{\text{ham}}(M)$$



$$\# \quad 0 \rightarrow H_{\text{dr}}^{m-2}(M) \rightarrow J^{m-2}(M) \hookrightarrow \mathcal{A}^{m-3}(M) \rightarrow \mathcal{X}_{\text{ex}}(M) \rightarrow 0$$

$$Sp(2n) \subset M_{2n \times 2n}^{\text{sym}}(R) \hookrightarrow O(m)$$

$$gl(n)^* \xleftarrow{J_L} M_x \xrightarrow{J_R} gl(m)^*$$

Identified  $\rightarrow$  Dunkl

$$\begin{array}{c} (Emb(S, M), \eta) \\ \swarrow \quad \searrow \\ \mathcal{C}(M)^* \quad \mathcal{X}_{\text{ve}}(S)^* = \Lambda^1(S) / \mathcal{A}^1(S) \end{array}$$

Non-exact symplectic

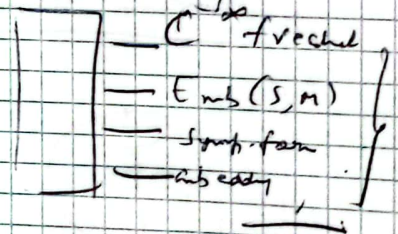
$$(H^1(S) = 0)$$

$$H^{m-2}(S) \rightarrow \mathcal{X}$$

$$e(M) : \mathcal{X}_{\text{ham}}(S) \hookrightarrow \mathcal{A}^{m-1}(S) \rightarrow \mathcal{X}_{\text{ex}}(S)$$

$\rightarrow$

$$\boxed{D = ?}$$



$$\delta_t v + \mathcal{L}_u v = -d\tilde{F}. \quad (1\text{-form})$$

$$\delta_t \omega + \mathcal{L}_u \omega = 0 \quad (2\text{-form})$$

$$u = T_g \omega_j$$

$$\text{where, } T_g = -g^{-1} \cdot \Delta_g^{-1} \operatorname{div}_g.$$

$\rightarrow$  Momentum conservation (space-derivative operator),  
 $\rightarrow$  Hodge conservation (slope, modified).

*(scribbled out text)*

